The effect of spatial scale on bias in regression models with spatial confounding

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Outline

1. Framework
   - Correlated Residuals
   - Spatial Confounding
2. Results
   - Analytic
   - Simulation
3. Extensions

Themes

- Intuition about residual correlation can be deceptive
- Scales of spatial correlation are critical
- Accounting for spatial correlation may help reduce bias from confounding in **some** situations.

Uncertainty and Correlated Residuals

- Variance of regression estimates, Var($\hat{\beta}$):
  - naive OLS variance is incorrect
  - GLS is the minimum variance estimator
    - lower variance than OLS with corrected variance estimate
- Question: How does residual correlation affect variance?
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Uncertainty and Correlated Residuals

- Variance of regression estimates, Var($\hat{\beta}$):
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  - GLS is the minimum variance estimator
    - lower variance than OLS with corrected variance estimate
- Question: How does residual correlation affect variance?
- Conventional wisdom: Correlated residuals reduce the effective sample size, so their presence adds uncertainty.

Uncertainty and Correlated Residuals (2)

- Reality:
  - Correlated residuals offer an **opportunity** to improve precision by systematically explaining a portion of the residual variability.
  - Equivalent models

$$\text{GLS:} \quad Y \sim \mathcal{N}(X\beta, \sigma_r^2 R + \tau^2 I)$$

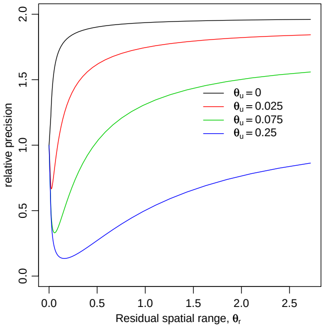
$$\begin{aligned} \text{GAM:} \quad Y &\sim \mathcal{N}(X\beta + g, \tau^2 I) \\ g &\sim \mathcal{N}(0, \sigma_r^2 R) \end{aligned}$$

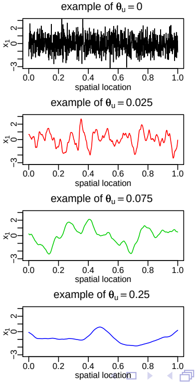
  - Heuristic is that fitting either a GLS or GAM model allows one to attribute residual variability to the spatial component of the residual, reducing the unexplained variability in the model and decreasing $\text{Var}(\hat{\beta})$.

Precision with Correlated Residuals

$$E(Var(\hat{\beta})^{-1}) = E(X_1^T \Sigma^{-1} X_1) = \mathrm{tr}(\Sigma^{-1}\sigma_u^2 R(\theta_u)) = \frac{\sigma_u^2}{\tau^2}\mathrm{tr}((I + \frac{\sigma_r^2}{\tau^2} R(\theta_r))^{-1} R(\theta_u)).$$

Results depend on the scale of the correlation in X_1 and the residual.





Key Question

- We know that attributing variability to a spatial component in the residual can reduce variance.
- Can it alleviate bias from an unmeasured, but spatially-correlated, confounder?
  - Potential mechanism: attribute variability from confounder to the spatial residual (or to a spatial term in the mean).
- Conventional Wisdom?
  - Accounting for spatial correlation in the residual can account for a spatial confounding and reduce (eliminate?) bias.
- Reality:
  - It depends on the spatial scales involved.
  - Dominici et al. (2004, JASA): control for spatial structure at large scales to eliminate confounding at that scale.
  - Goal is to assess association based on nearby observations, which share the same large-scale spatial effect.

Thought Experiment

- Suppose pollution varies smoothly in space. Also, suppose that (unmeasured) SES varies smoothly in space.
- If we analyze a health outcome as a function of pollution, the residuals will be correlated because of SES.
- There is a fundamental non-identifiability in the model

$$Y_i = X(s_i)\beta + g(s_i) + \epsilon_i$$

  which we could re-express as

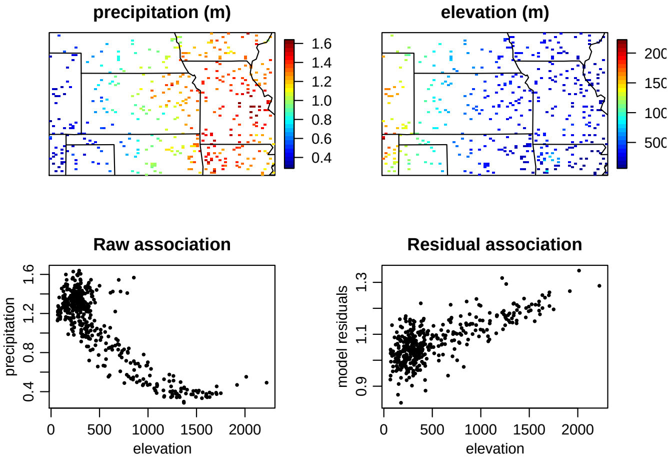
$$Y_i = g^*(s_i) + \epsilon_i.$$

  That is, how do we separate the pollution effect from the spatial effect (spatial confounder) if the pollution effect is just another form of spatial effect.
- Question: how does the model attribute variation between $X(s)\beta$ and $g(s)$?

Scale Matters

- A non-health example: how does elevation affect precipitation in the central United States?
- At large scale, precipitation increases with decreasing elevation as topography slopes gently downwards from the Rockies to the Mississippi River.
  - Elevation is not the causal effect.
- At smaller scale, precipitation increases with increasing elevation.
- A spatial model here can account for confounding from other factors that vary smoothly west to east, and isolate the elevation effect to the effect of elevation at small scales.

Association of Elevation and Precipitation



A Simple Model

- We can explore bias by starting with a simple generative model:

$$y_i = \beta_1 x_1(s_i) + \beta_2 x_2(s_i) + \epsilon_i$$

  Let $x_1(s)$ and $x_2(s)$ be Gaussian processes, with $\text{Cor}(x_1(s_i), x_2(s_i)) = \rho$.
- If x_2 is unmeasured, we arrive at the GLS model

$$y_i = \beta_1 x_1(s_i) + \epsilon_i^*$$
$$\text{Cov}(\epsilon^*) = \Sigma = \sigma_r^2 R(\theta_r) + \tau^2 I$$

  where $\sigma_r^2 = \beta_2^2 \text{Var}(x_2)$.
- Bias comes from fitting models under the assumption that ϵ^* is uncorrelated with x_1.

Known parameters, single scale

- Suppose $x_1(s)$ and $x_2(s)$ share the same range of spatial correlation, but may be scaled differently in magnitude, namely, $\text{Cov}(x_1) = \sigma_c^2 R(\theta_r)$ and $\text{Cov}(\beta_2 x_2) = \beta_2^2 \sigma_2^2 R(\theta_r)$, then

$$\begin{aligned} E(\hat{\beta}_1|x_1) &= \beta_1 + (x_1^T \Sigma^{-1} x_1)^{-1} x_1^T \Sigma^{-1} E(x_2|x_1)\beta_2 \\ &= \beta_1 + \rho \frac{\sigma_2}{\sigma_c} \beta_2 \end{aligned}$$

  because $E(x_2|x_1) = \rho\sigma_2\sigma_c R(\theta_r)\sigma_c^{-2} R(\theta_r)^{-1} x_1$.

- The resulting bias, $\rho\frac{\sigma_2}{\sigma_u}\beta_2$, is the same as if the covariates were not spatially structured.
- Heuristically, the model attributes variability from the confounder to the covariate of interest.

Known parameters, multi-scale

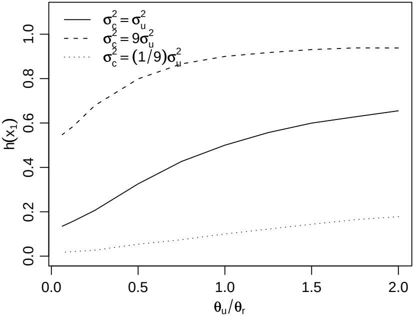
Let $x_1(s) = x_c(s) + x_u(s)$ with $\text{Cov}(x_1) = \sigma_c^2 R(\theta_r) + \sigma_u^2 R(\theta_u)$.
Let $\text{Cov}(x_2) = \sigma_2^2 R(\theta_r)$ and $\text{Cor}(x_c(s_i), x_2(s_i)) = \rho$.

$$E(\hat{\beta}_1|x_1)$$

$$= \beta_1 + (x_1^T \Sigma^{-1} x_1)^{-1} x_1^T \Sigma^{-1} E(x_2|x_1)\beta_2$$

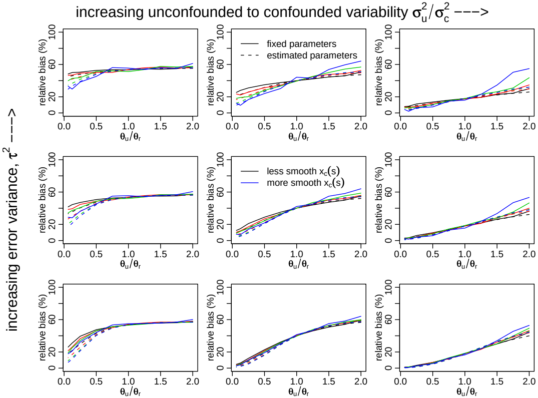
$$= \beta_1 + \frac{x_1^T \Sigma^{-1}(I + \frac{\sigma_u^2}{\sigma_c^2} R(\theta_u) R(\theta_r)^{-1})^{-1} x_1}{(x_1^T \Sigma^{-1} x_1)} \rho \frac{\sigma_2}{\sigma_c} \beta_2$$

$$= \beta_1 + h(x_1)\rho \frac{\sigma_2}{\sigma_c} \beta_2$$



Unknown parameters

Simulation results indicate that bias when estimating parameters in a GLS framework (or also in a GAM framework) is similar to that with known parameters.



Heuristics

- Reducing bias requires the covariate of interest to have a spatial scale at which it is unconfounded, and that scale must be smaller than the scale at which confounding operates.
- We would like the covariate to have as much variation at the unconfounded scale and as little at the confounded scale as possible.
- Other results are straightforward and match the non-spatial setting for confounding. We want:
  - the magnitude of variation in the confounder (or its effect on the outcome) be small.
  - the correlation between confounder and covariate to be small.

Ongoing work

- Analysis of precision and MSE
- Simulations for non-linear settings
- Effects of choosing incorrect parameter values to minimize bias
    - Using fixed df to model the residual correlation (a la Dominici et al. 2004)
- Areal data settings
- Implications of measurement error

Areally-aggregated Data

- Aggregated data in areal units such as zip codes, census tracts and counties are often the finest resolution data available for disease mapping analyses.
- Spatial confounding may be an issue in spatial regression models for aggregated data.
- Conditional auto-regressive (CAR) models are often used; these models smooth based on weighted averaging of neighboring units.
- Two key issues in areal models:
    1. Aggregation smooths over fine-scale heterogeneity.
    2. CAR models (by using local averaging) do not model large-scale spatial patterns.
- Both of these issues suggest that bias could be substantial in CAR-type models based on the results presented here.

Measurement Error

- Classical error:
  - Preliminary work suggests that under classical error, the model attributes variability in the outcome to the spatial residual, not to the error-contaminated covariate of interest.
  - Model attenuates the effect estimate because the spatial residual is a well-measured surrogate that can stand in for the covariate.
- Berkson error/regression calibration:
  - Gryparis, Paciorek, and Coull (under revision) argue that spatial smoothing models are a form of regression calibration that induce Berkson type error when using predictions
  - Under Berkson error, we should be in the framework discussed here, except that smoothing done to make predictions will reduce fine-scale heterogeneity, decreasing our ability to reduce bias.