

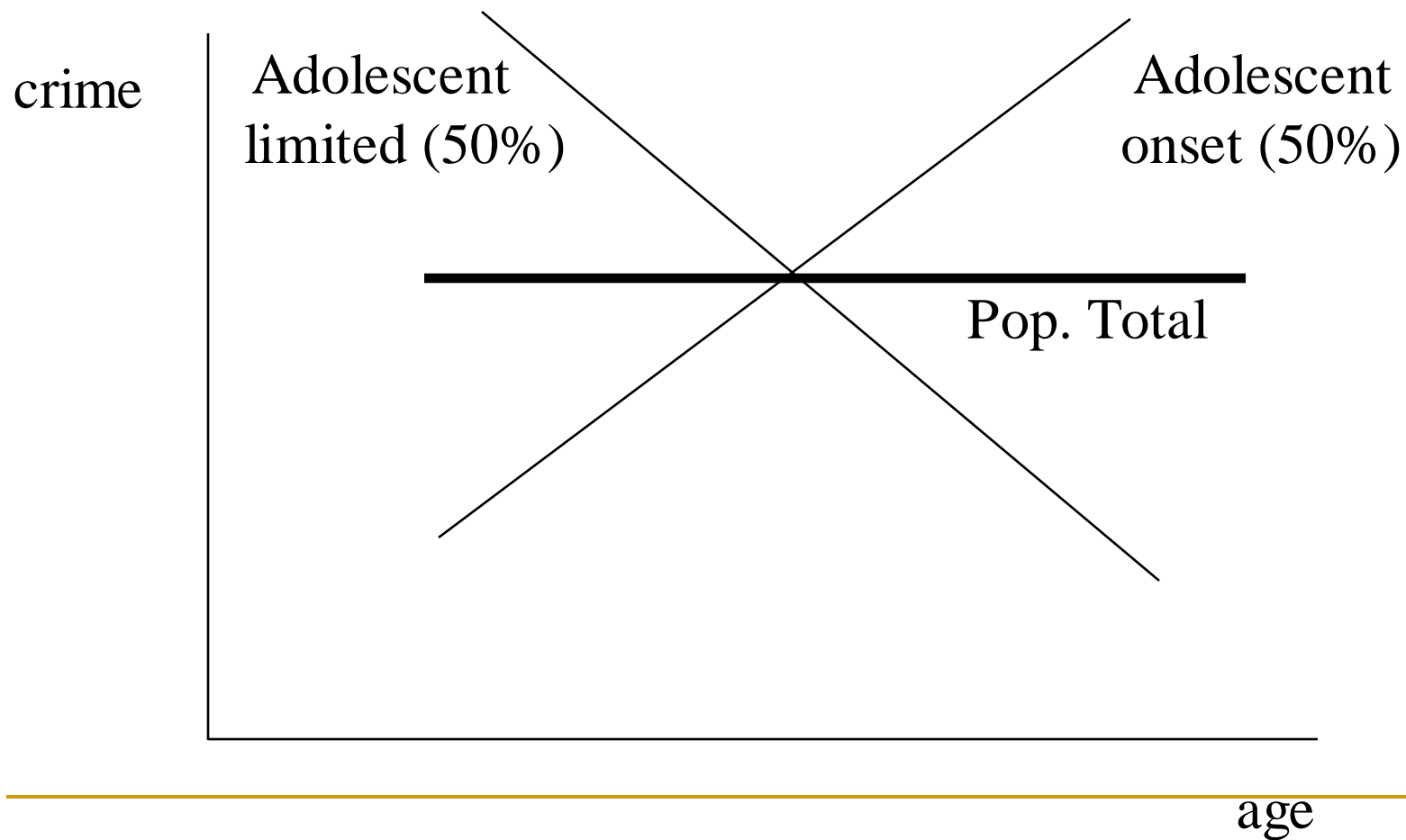
---

# Proc Traj: A SAS Procedure for Group Based Modeling of Longitudinal Data

---

Bobby L. Jones, PhD  
Carnegie Mellon University

## An Illustrative Example



---

## Proc Traj

- SAS procedure that identifies clusters of individuals following similar progressions of an outcome over time, age, ... , by fitting a group based model
- Proc Traj is used like any SAS proc e.g. Proc Glm, Proc Mixed ...
- Traj download, online documentation, examples, and installation information at <http://www.andrew.cmu.edu/~bjones>

---

# Proc Traj

- Easy-to-use, PC SAS procedure
- Handles missing data
- Handles sample weights
- Allows irregular spacing of measurements
- Accommodates over-lapping cohort designs

---

## Basic Proc Traj Model Estimates

- Number of groups via Bayesian Information Criterion (BIC) model comparisons
- Group mean shapes
- Group sizes
- Individual probabilities of group membership
- Effect sizes for time-independent (risk) and time-dependent variables (tcov)

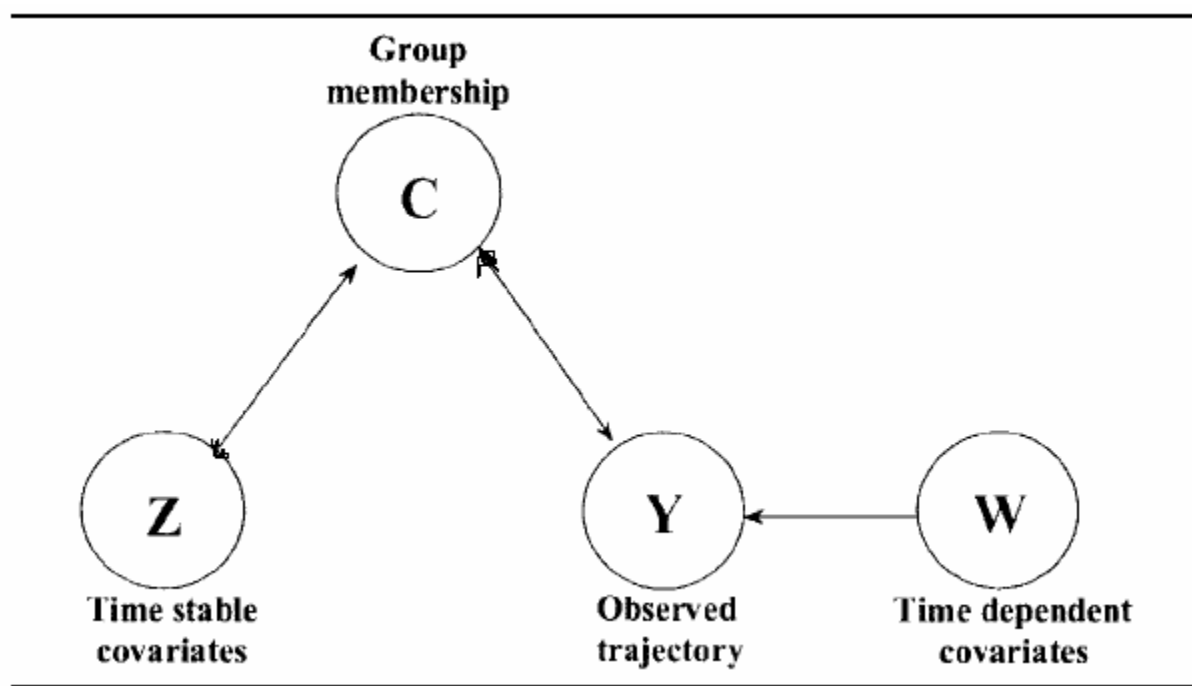
$$BIC = \log(L) - .5 * \log(n) * k$$

$$L = \log \textit{likelihood}$$

$$n = \textit{sample size}$$

$$k = \textit{number of parameters}$$

# Model Form and Independence Assumptions



---

# Proc TRAJ Basic Poisson Model Example with Confidence Limits

```
PROC TRAJ DATA=A OUT=OF OUTPLOT=OP OUTSTAT=OS CI95M;  
    VAR O1-O11;          /* Conviction Variables      */  
    INDEP T1-T11;        /* Age Variables            */  
    MODEL ZIP;           /* Poisson Model           */  
    ORDER 0 2 2 2;      /* Flat and Quadratic for Four Groups */  
RUN;
```

```
%TRAJPLOTNEW (OP, OS,,,"Annual Conviction Rate","Age");
```

411 subjects from working-class section of London (Farrington and West, 1990)  
Criminal offense convictions recorded annually from age 10 – age 30

# Proc Traj Output

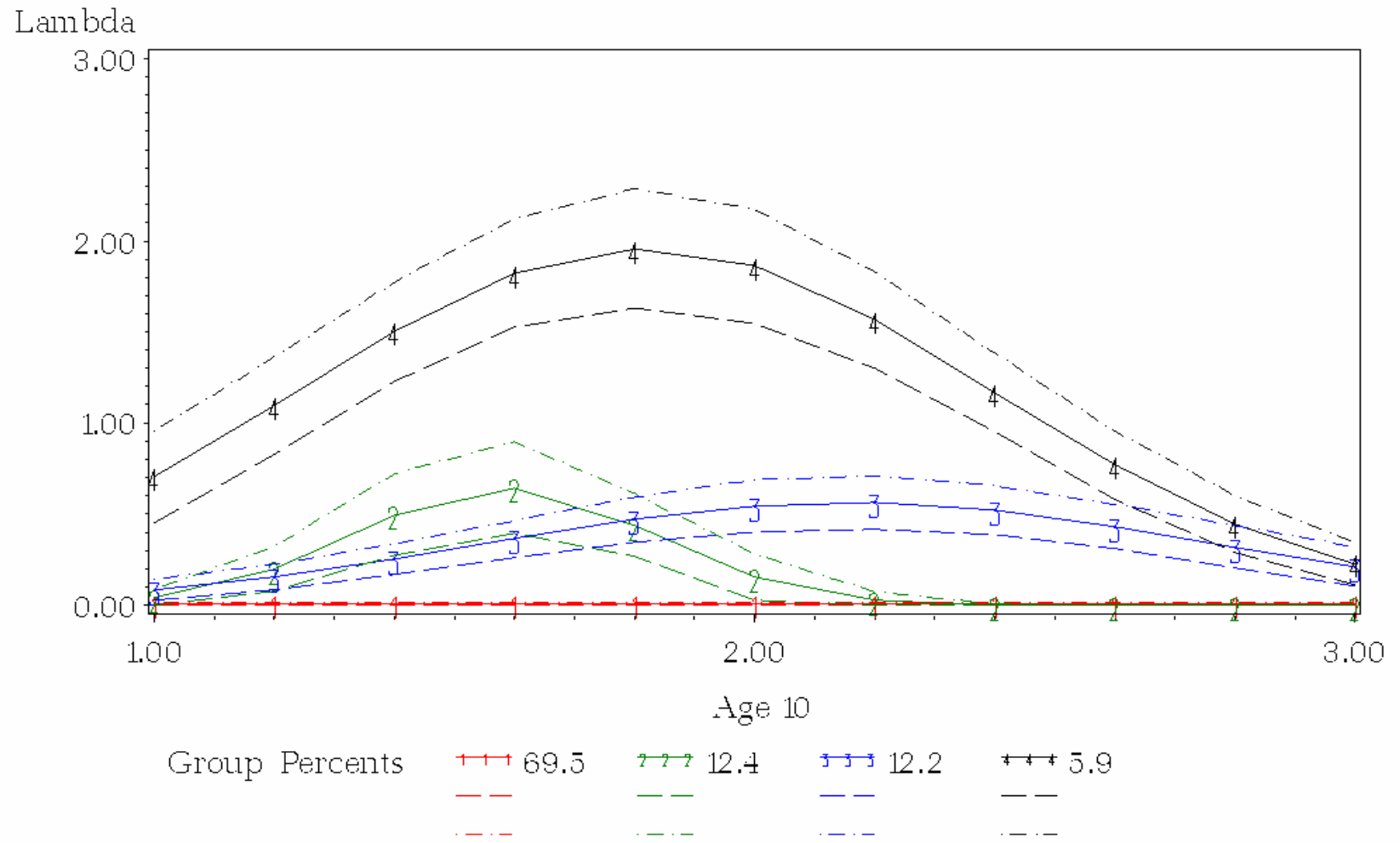
1	Intercept	-4.57337	0.27345	-16.724	0.0000
2	Intercept	-20.49791	4.15470	-4.934	0.0000
	Linear	25.38542	5.47675	4.635	0.0000
	Quadratic	-8.02886	1.78662	-4.494	0.0000
3	Intercept	-7.00478	1.16221	-6.027	0.0000
	Linear	5.95561	1.15414	5.160	0.0000
	Quadratic	-1.37783	0.27769	-4.962	0.0000
4	Intercept	-4.36648	0.75693	-5.769	0.0000
	Linear	5.54530	0.81271	6.823	0.0000
	Quadratic	-1.52575	0.21040	-7.252	0.0000
	Group membership				
1	(%)	69.48667	3.17546	21.882	0.0000
2	(%)	12.40384	2.72478	4.552	0.0000
3	(%)	12.19109	2.17877	5.595	0.0000
4	(%)	5.91840	1.29345	4.576	0.0000

BIC= -1516.57 (N=403) BIC= -1532.15 (N=4433) AIC= -1490.57



# Trajectories of Convictions

(Criminology (1993.1995))



---

# Short live demo

- Use of Sas Explorer to look at files
- Example 1-Physical Aggression Trajectories
- Montreal Data: 1037 Caucasian, francophone, nonimmigrant males
- First assessment at age 6 in 1984

---

# Types of Data Handled

- Psychometric scale data--censored normal (tobit) model
- Count data--poisson-based model
- Binary data--logit-based model
- Normally distributed data (set min and max outside the range of the data)

---

## The Likelihood Function

$Y_i$  = trajectory data for subject  $i$

$$P(Y_i) = \sum_j \pi_j(x_i) P^j(Y_i)$$

$P^j(Y_i)$  = probability of  $Y_i$ , if belonging to group  $j$

$\pi_j(x_i)$  = probability of belonging to group  $j$  for covariates (risk)  $x_i$

$$\pi_j(x_i) = \frac{e^{x_i \theta_j}}{\sum e^{x_i \theta_j}}$$

$$L = \prod_{i=1}^N P(Y_i).$$

---

# Zero-inflated Poisson Model for Count Data

IORDER used to specify alpha polynomials

$$p(x) = \begin{cases} 0 \text{ with probability } \rho \\ \text{Poisson}(\lambda) \text{ with probability } 1 - \rho \end{cases}$$

$$\ln(\lambda) = \beta_0 + \beta_1 \text{age} + \beta_2 \text{age}^2 + \beta_3 \text{age}^3$$

$$\rho = \frac{e^{\alpha_0 + \alpha_1 \text{age} + \alpha_2 \text{age}^2 + \alpha_3 \text{age}^3}}{1 + e^{\alpha_0 + \alpha_1 \text{age} + \alpha_2 \text{age}^2 + \alpha_3 \text{age}^3}}$$

---

# Logit Model for Binary Data

$$P(y = 1) = \frac{e^{\beta_0 + \beta_1 \text{age} + \beta_2 \text{age}^2 + \beta_3 \text{age}^3}}{1 + e^{\beta_0 + \beta_1 \text{age} + \beta_2 \text{age}^2 + \beta_3 \text{age}^3}}$$

$$P(y = 0) = 1 - P(y = 1)$$

# Censored Normal (CNORM) Model

$\Pr(Y_i = y_i \mid C_i = k, \mathbf{W}_i = \mathbf{w}_i) =$

$$\prod_{y_{ij} = \min} \Phi\left(\frac{\text{Min} - \mu_{ijk}}{\sigma}\right) \prod_{\text{Min} < y_{ij} < \text{Max}} \frac{1}{\sigma} \phi\left(\frac{y_{ij} - \mu_{ijk}}{\sigma}\right) \prod_{y_{ij} = \text{Max}} \left(1 - \Phi\left(\frac{\text{Max} - \mu_{ijk}}{\sigma}\right)\right),$$

where

$$\mu_{ijk} = \beta_{0k} + \text{age}_{ij} \beta_{1k} + \text{age}_{ij}^2 \beta_{2k} + \dots + w_{ij} \delta_k.$$

Trajectory mean calculation complicated!  
Use plot file for trajectory means (for all models)

---

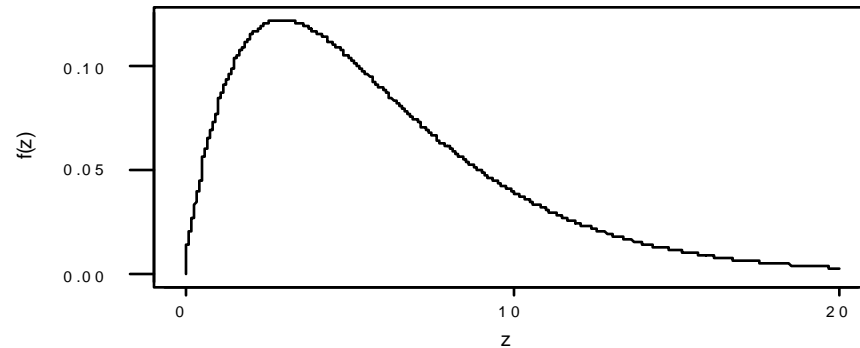
## Group-Based Trajectory Modeling Compared to Conventional Growth Curve Modeling (HLM)

- Common point of departure: both model individual level trajectories by a polynomial equation in age or time:  $\beta_0 + \beta_1 Age + \beta_2 Age^2$
- Point of departure: how to model individual-level differences in developmental trajectories (e.g., population heterogeneity)
  - HLM use normally distributed random effects
  - Group-based trajectory can approximate an unknown distribution of individual differences with groups

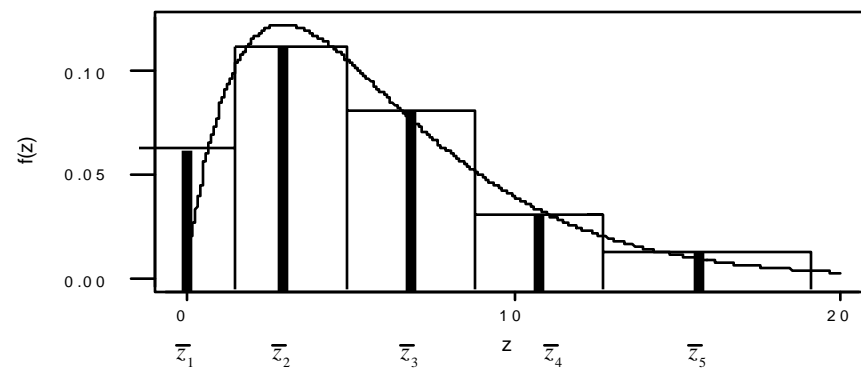


# Groups Can Approximate an Unknown Distribution

Panel A



Panel B



---

# Advances In Proc Traj

---

---

# Missing Trajectory Data

- Intermittently Missing
  - Maximum Likelihood Robust Here
- Subject Dropout
  - Model Probability of Dropout on Previous Measurements and Other Variables

---

## Likelihood Function Modified for Dropout

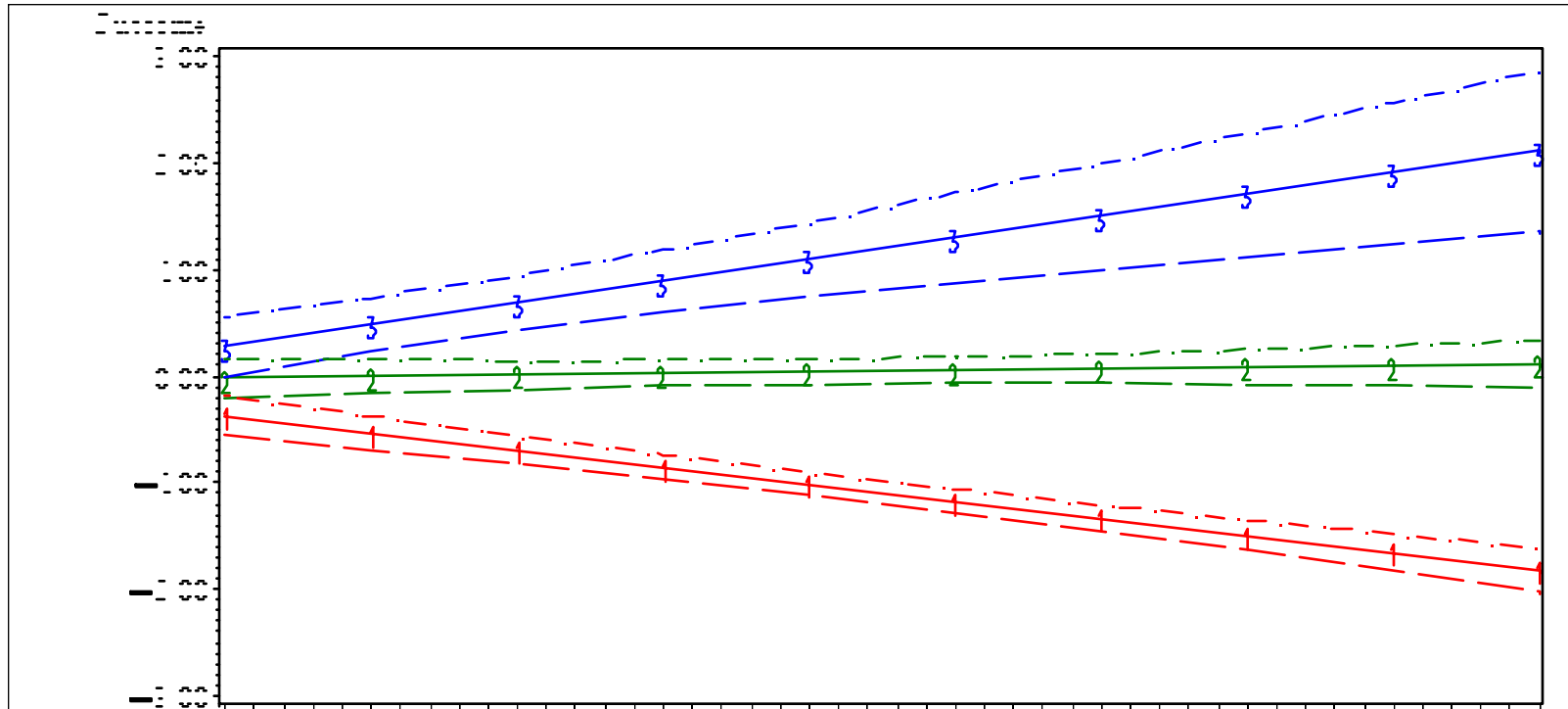
$$P(Y_i) = \sum_j \pi_j(x_i) P^j(Y_i) P^j(D_i | Y_i)$$

$P^j(Y_i)$  = probability of  $Y_i$ , for group  $j$

$P^j(D_i | Y_i)$  = probability of *Dropout*, for group  $j$  as a function of measurements before dropout.

$$P(D_i | Y_i) = \frac{e^{\beta_0 + \beta_1 y_{t-1} + \beta_2 y_{t-2}}}{1 + e^{\beta_0 + \beta_1 y_{t-1} + \beta_2 y_{t-2}}}$$

# Data w/ Dropout (N=200: 25%, 50%, 25%)



	1	2	3	4	5	6	7	8	9	10	Didn't Dropout
G1	0	0	0	0	1	0	0	1	0	0	48
G2	2	3	7	6	8	5	7	1	3	6	52
G3	1	8	7	8	7	8	1	3	6	1	0

---

# Estimated Group Percentages

- Group size estimates (no dropout modeling)

- Bias in group size

1 (%)  $30.9 \pm 4.2$

2 (%)  $53.3 \pm 5.4$

3 (%)  $15.8 \pm 4.5$

With dropout modeled no bias in group size

1 (%)  $27.1 \pm 3.7$

2 (%)  $45.9 \pm 7.3$

3 (%)  $27.0 \pm 7.1$

- But no bias in trajectory shape estimates with or without dropout modeled

---

# Proc Traj Syntax for Dropout Model

```
proc traj data=a out=b outstat=os outplot=os;  
var opp1-opp5; indep t1-t5;  
model cnorm; max 10; order 0 2 2;  
dropout 2 2 2;  
run;
```

---

# Missing Covariate Data

- Default is complete case analysis
- Alternative: multiple imputation
  - PROC MI (produce replicate data sets)
  - PROC MIANALYZE (analyze results)



---

# Multitrajectory Modeling

- Extend model to more than two repeated measures.
- Each group is now defined by multiple trajectories: one for each response of interest.

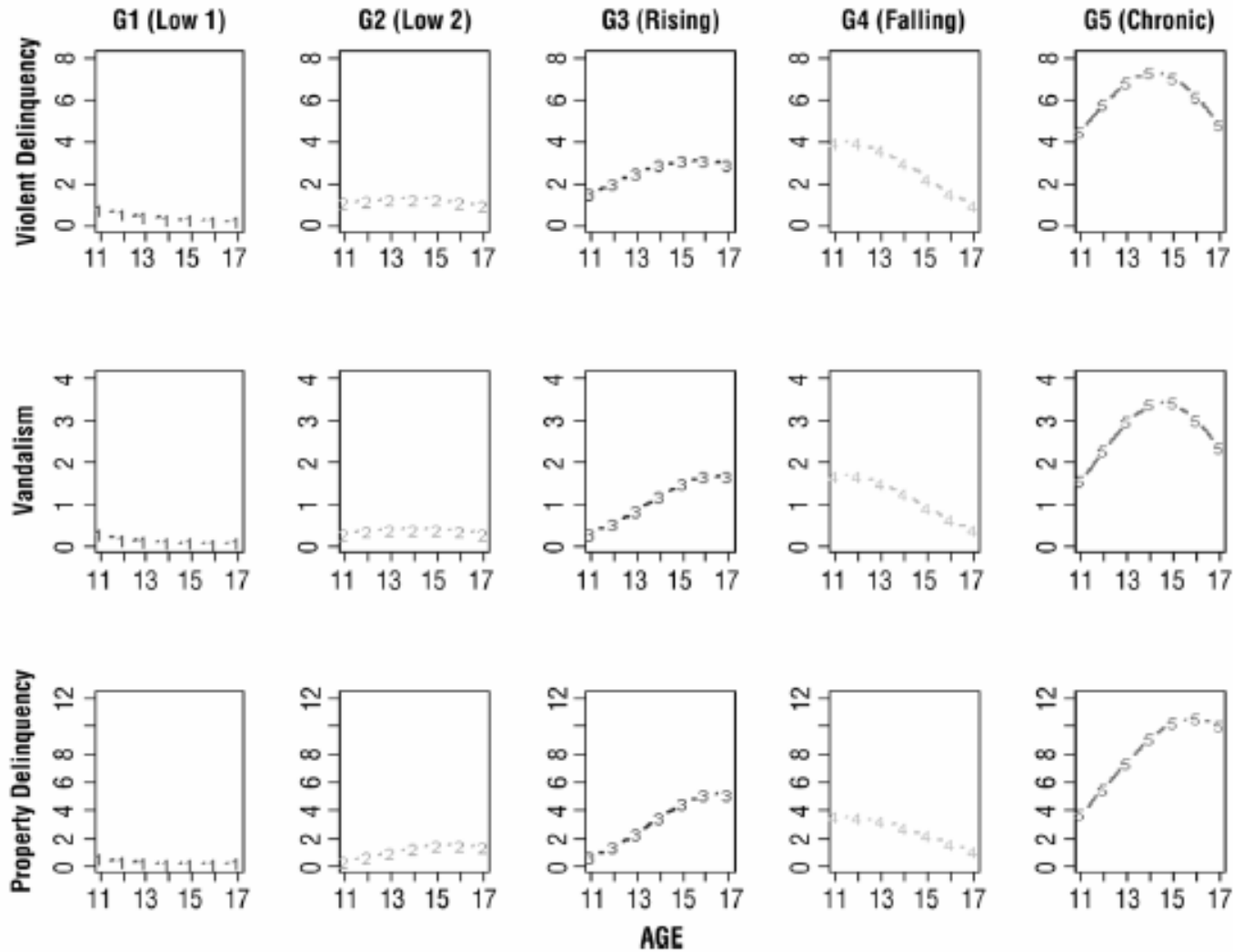
---

## Proc Traj Syntax for Multitrajectory Model

```
proc traj data=montreal outplot=op outstat=os
outplot2=op2 outstat2=os2 outplot3=op3 outstat3=os3;
var bat89-bat95; indep t1-t7; model zip;
order 2 2 2 2 2;
var2 det89-det95; indep2 t1-t7; model2 zip;
order2 2 2 2 2 2;
var3 vol89-vol95; indep3 t1-t7; model3 zip;
order3 2 2 2 2 2;
multigroups 5;
run;
```

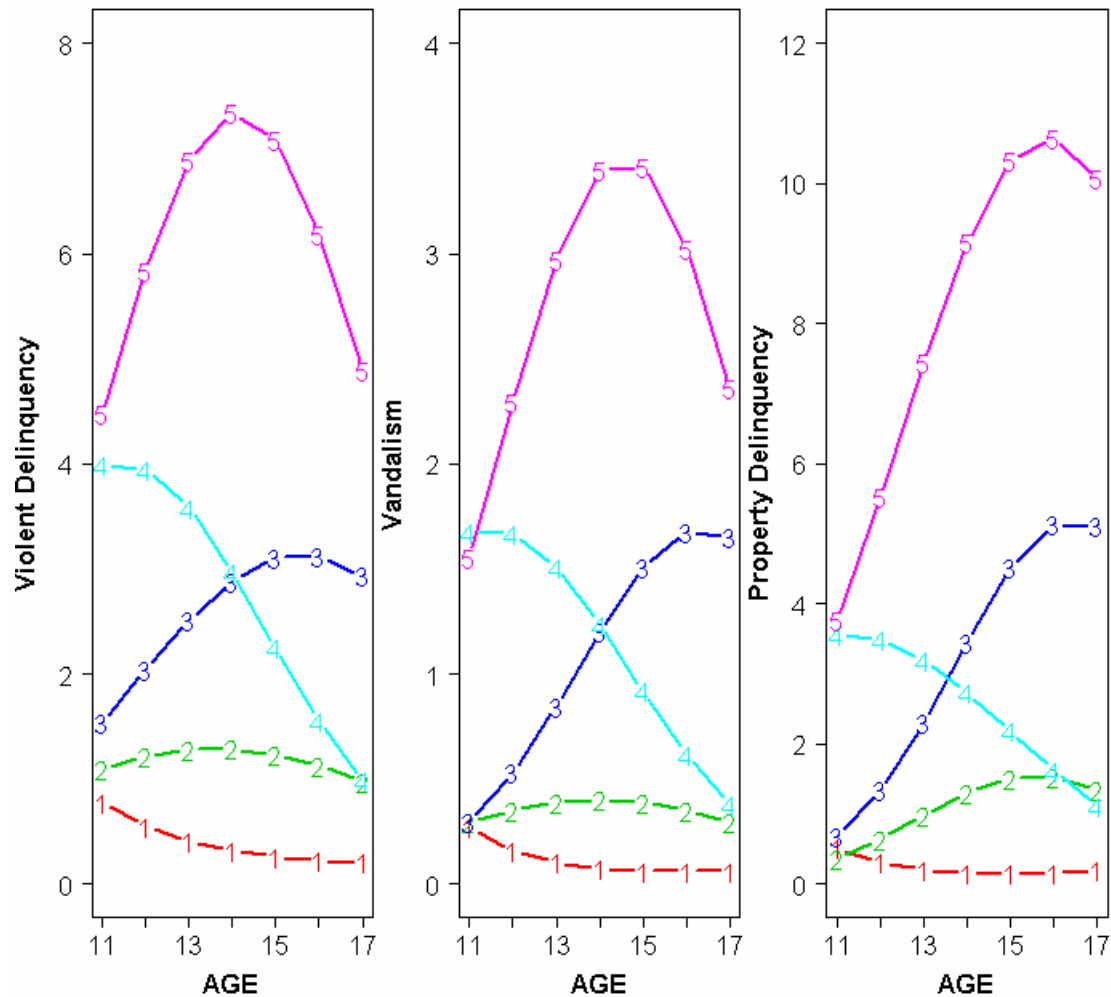
---

# Five Group Multiple Trajectory Model



# Five Group Multiple Trajectory Model

Figure 5B: Trajectory Plots Organized by Behavior



---

## Relating Trajectory Groups to a Subsequent Outcome Variable

- Link trajectory groups with an outcome variable, measured once on or after the termination of the trajectory.
- Example: Relating trajectories of opposition from age 6 to 13 to average number of sexual partners at age 14.

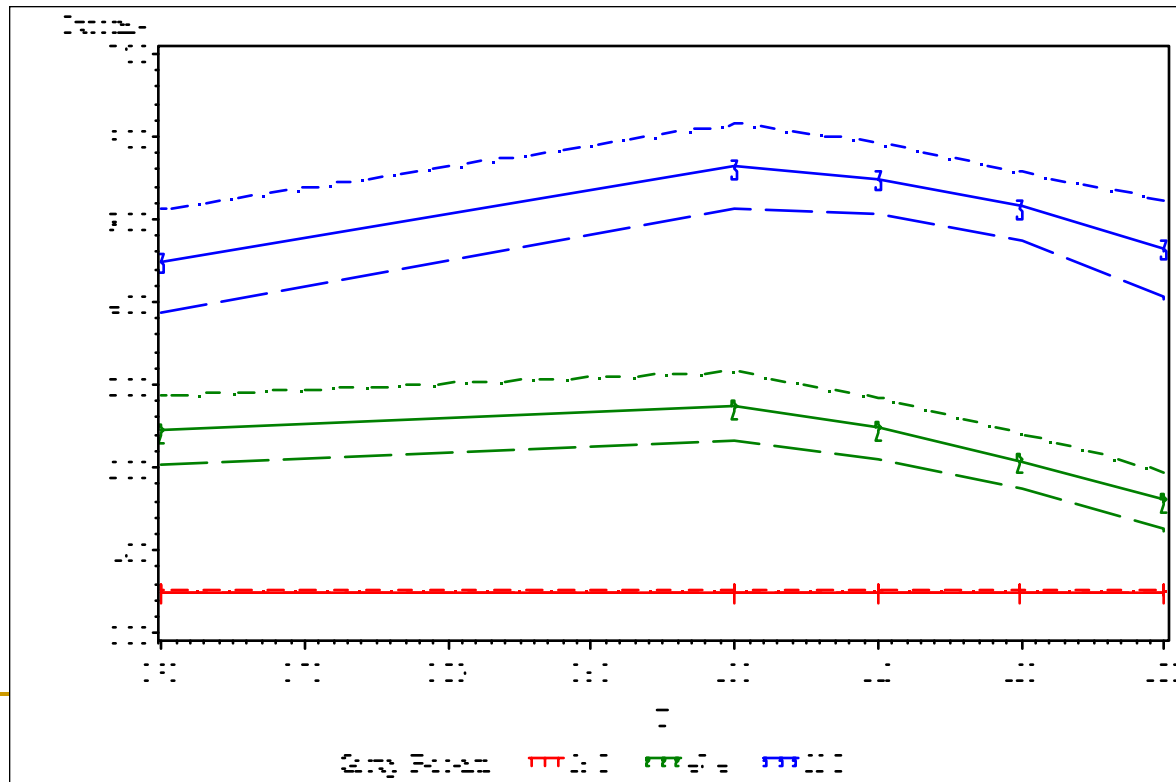
---

# Proc Traj Syntax

```
proc traj data=a out=b outstat=os outplot=os  
  outcomeresults=or;  
var opp1-opp5; indep t1-t5;  
model cnorm; max 10; order 0 2 2;  
outcome nbp14;  
omodel poisson;
```

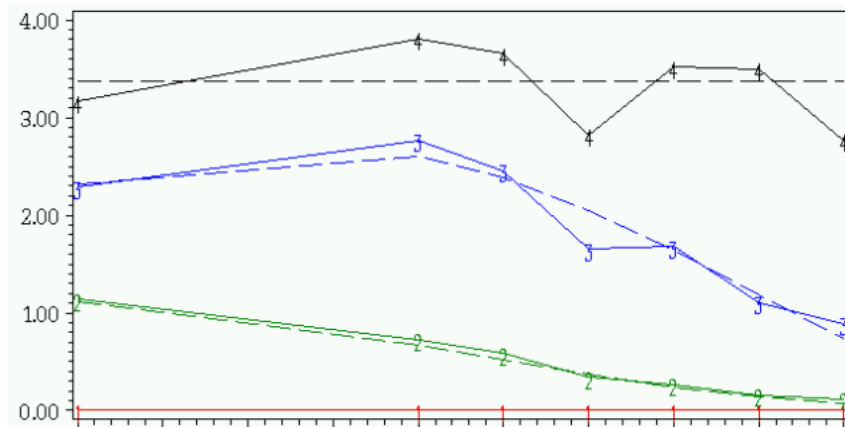
# Outcome Analysis (opp 8-13, nbp14)

Gp	L95 Mean	Mean Outcome	U95 Mean
1	0.27	0.35	0.44
2	0.59	0.68	0.78
3	1.48	1.72	2.00



# Setting Risk Factors to Calculate Effect on Group Membership

- Physical Aggression Trajectory Model
- Risk Factors
  - Broken Home at Age 5
  - Low IQ
  - Low Maternal Education
  - Mother Began Childbearing as a Teenager





---

# Setting Risk Factors

```
data predict;  
input nint5 lowiq edmom teenmom;  
cards;  
0 0 0 0  
1 0 0 0  
0 1 0 0  
1 1 1 1  
run;
```

```
data combine;  
set predict originalDat;  
run;
```

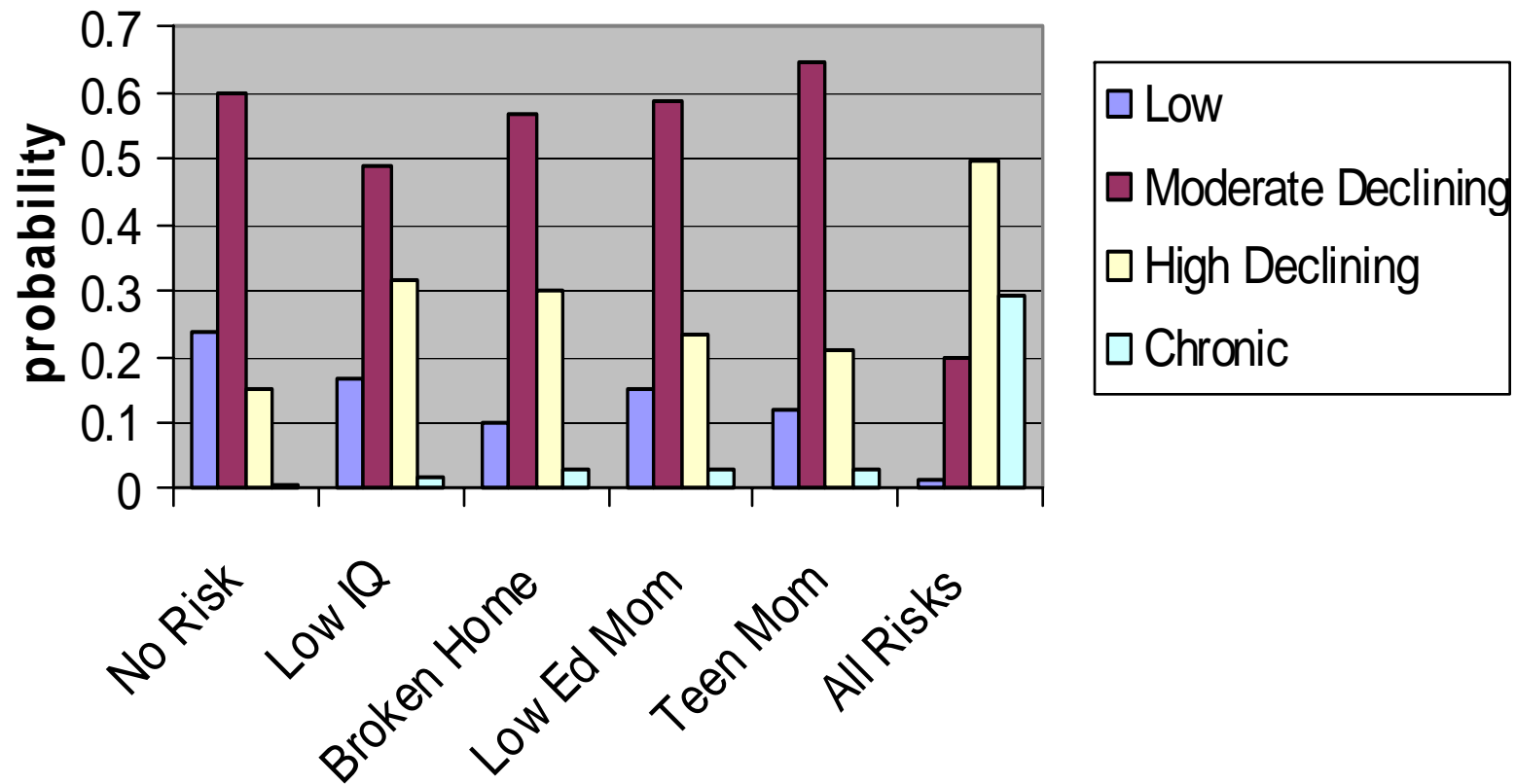
---

# Results in Outfile

## nint5 lowiq edmom teenmom

RISK	G1Prob	G2Prob	G3Prob	G4Prob
0 0 0 0	0.237612	0.599554	0.154430	0.008404
1 0 0 0	0.102632	0.570928	0.298253	0.028187
0 1 0 0	0.171699	0.490363	0.317862	0.020076
1 1 1 1	0.009400	0.196002	0.498584	0.296015

## Impact of Risk Factors on Group Membership Probabilities



---

# %trajtest

- The trajtest macro tests hypotheses about model parameters
- For linear hypothesis, test based on  $(Lb - c)' (S')^{-1} (Lb - c)$  (Wald test)  
 $c_1 \times \text{variable}_1 \pm c_2 \times \text{variable}_2 \pm \dots \pm c_n \times \text{variable}_n$
- Variable names correspond to names in the OUTEST= file.
- %trajtest('linear1=linear2=linear3')
- %trajtest('linear1=linear2,quadra1=quadra2')
- %trajtest('risk1=risk2=risk3')

---

# Refgroup;

- Allows specification of reference group for risk factor analysis.
- Refgroup 2;
- Demo

---

## Readings (Jones articles available from Traj website)

- Jones, B., D.S. Nagin. And K. Roeder. 2001. "A SAS Procedure Based on Mixture Models for Estimating Developmental Trajectories." *Sociological Research and Methods*, 29: 374-393.
- Jones, B. and D.S. Nagin. 2007. "Advances in Group-based Trajectory Modeling and a SAS Procedure for Estimating Them," *Sociological Research and Methods*, 35: 542-571.
- Nagin, D. S. 2005. *Group-based Modeling of Development*. Cambridge, MA.: Harvard University Press.
- Nagin, D.S. and R. E. Tremblay. 2005. "Developmental Trajectory Groups: Fact or a Useful Statistical Fiction?." *Criminology*, 43:873-904.
- Nagin, D. S., and R. E. Tremblay. 2001. "Analyzing Developmental Trajectories of Distinct but Related Behaviors: A Group-based Method." *Psychological Methods*, 6(1): 18-34.
- Nagin, D. S. 1999. "Analyzing Developmental Trajectories: A Semi-parametric, Group-based Approach." *Psychological Methods*, 4: 139-177.